Characterization of posets connected to preference structures

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Abstract

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Weakly orthocomplemented posets are introduced and investigated in connection to poset-valued preference structures, as a generalization of orthocomplemented posets. Different representations of such posets are presented. Moreover, necessary and sufficient conditions for a pointwise closure system to be a system of cuts of poset valued preference structures with a fixed co-domain is given. Also we give a convenient representation of posets via join-irreducibles in the framework of poset valued intuitionistic structures.

Orthocomplemented poset

An **orthocomplemented poset** $(P, \leq, ^{\perp}, 0, 1)$ is a poset $(P, \leq)$, equipped with the top 1 and a bottom 0, and with an antitone involution $^{\perp}$ over P such that for all $x \in P$, the join $x \vee x^{\perp}$ exists and $x \vee x^{\perp} = 1$.

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Two elements $x, y \in P$ are called **orthogonal** if $x \leq y^\perp$.

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Then P is called an **orthogonal poset** [I. Chajda, 2014].

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Examples of weakly orthocoplemented posets are Boolean lattices, orthcomplemented posets, orthogonal posets, bounded chains.

Examples

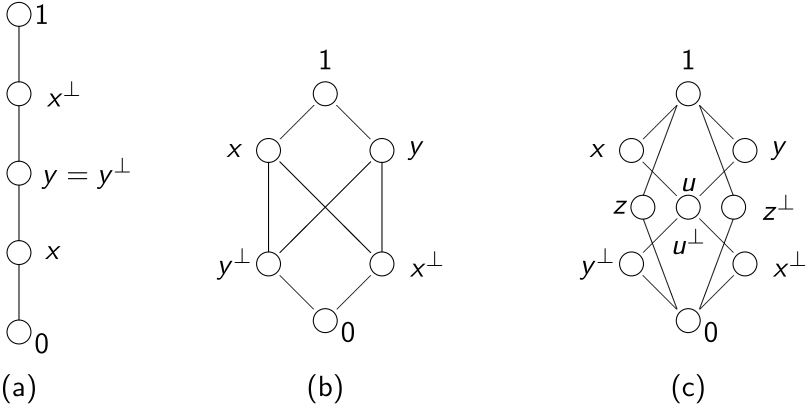

Figure 3

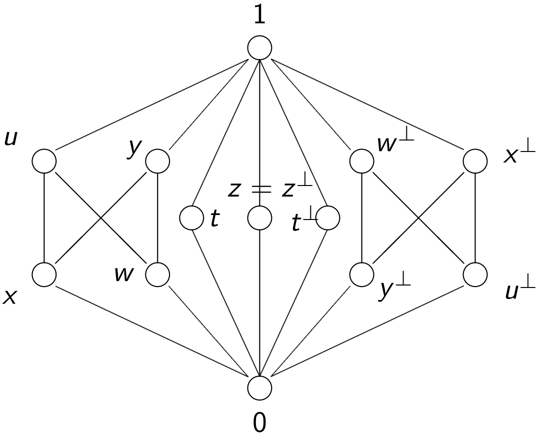

Figure 4

Preliminaries

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If $\mu : X \to P$ is a P-valued set on X then, for $p \in P$, the set

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P-valued relations

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(a) If $p \leq q$, then $R_q \subseteq R_p$.

(b) For $a, b \in X$

$$R(a,b) = \bigvee\{p \in P \mid (a,b) \in R_p\}$$

(The join on the right exists in $(P, \leq)$ for all $a, b \in X$ and is equal to $R(a,b)$.)

(c) If $Q \subseteq P$, and there exists a supremum of Q ($\bigvee\{p \mid p \in Q\}$), then

$$\bigcap\{R_p \mid p \in Q\} = R_{\bigvee\{p \mid p \in Q\}}.$$

(d) The collection $\mathcal{R}_P$ is a centralized system.

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A preference relation on a set of alternatives A is often investigated within the framework of a preference structure - ordered triple (P, I, J), in which P is a strict preference, I indifference and J incomparability relation on A.
Associated to any classical preference structure without incomparable elements we could consider a three-valued binary relation R such that $R(a, b) = 1$ if $(a, b) \in P$, $R(a, b) = 1/2$ if $(a, b) \in I$ and $R(a, b) = 0$ if $(b, a) \in P$.

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A more realistic description of relations can be obtained if we consider R taking values on $[0, 1]$ instead of just $\{0, 1/2, 1\}$.
They are called probabilistic relations and they are applied in decision making, mathematical psychology, etc. They are often also called reciprocal or ipsodual relation.

Poset valued preference relations

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Let X be the set of alternatives. The mapping $R : X \times X \to [0,1]$ is a **reciprocal relation** on X if for any $a, b \in X$

$$R(a,b) + R(b,a) = 1$$

or, equivalently, $R(a,b) = R^c(b,a)$, where R^c denotes the complement of R ($R^c(x,y) = 1 - R(x,y)$).

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The definition above implies that for any $a \in X$, $R(a, a) = \frac{1}{2}$. Hence the value $\frac{1}{2}$ represents indistinguishability for $[0, 1]$-valued reciprocal relations.

Moreover, for any $a, b \in X$, one of the numbers $R(a, b)$, $R(b, a)$ is in the interval $[0, \frac{1}{2}]$, while the other is in $[\frac{1}{2}, 1]$.

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Hence, in this approach:

- $R(a,b) = 1/2$ indicates indifference between a and b,
- $R(a,b) = 1$ indicates that a is absolutely preferred to b, and
- $R(a,b) > 1/2$ indicates that a is preferred to b to some degree.

Preference relations

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Let X be a nonempty set (universe of objects) and P a bounded poset. The mapping $R : X \times X \to P$ is a **poset-valued reciprocal preference relation** (a P-valued preference relation) on X if for any $a, b, c, d \in X$,

$$R(a, b) \leq R(c, d) \text{ implies } R(d, c) \leq R(b, a) \text{ and}$$

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As an immediate consequence, we get that if the mapping $R : X \times X \to P$ is a P-valued preference relation, then for any $a, b, c, d \in X$ we have the following equivalence:

$$R(a, b) \leq R(c, d) \quad \textit{if and only if} \quad R(d, c) \leq R(b, a).$$

Preference relations

As another consequence we have the following: $R(a, b)$ is incomparable with $R(c, d)$ if and only if $R(b, a)$ is incomparable with $R(d, c)$.

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Lemma

Let X be a nonempty set and let P be a poset. If the mapping $R : X \times X \to P$ is a P-valued preference relation on X, then for all $a,b,c,d \in X$, we have that

$$R(a,b) = R(c,d) \Longleftrightarrow R(d,c) = R(b,a).$$

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For P-valued preference relations in general, there is no single value representing indistinguishability, corresponding to the value $\frac{1}{2}$ from the unit interval case. However, from the definition of a P-valued preference relation, we obtain a kind of a set of equilibria, as follows:

Preference relations

Lemma

Let X be a nonempty set and P a poset. If the mapping $R : X \times X \to P$ is a P-valued preference relation on X, then for any $a, b \in X$ the values $R(a, a)$ and $R(b, b)$ are either equal or incomparable.

Proposition

Let R be a poset valued preference relation on X. For any $a, b \in X$, if $R(a, b)$ and $R(b, a)$ are incomparable, then for every $c \in X$, both $R(a, b)$ and $R(b, a)$ are incomparable with $R(c, c)$. Further, if $R(a, b) \leq R(c, c)$ and $R(b, a) \leq R(d, d)$ for some $c, d \in X$, then $R(a, b) = R(b, a) = R(c, c) = R(d, d)$.

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As consequence, whenever $R(a, b)$ and $R(c, c)$ are comparable, then $R(c, c)$ is between $R(a, b)$ and $R(b, a)$, with respect to the order in P.

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As consequence, whenever $R(a, b)$ and $R(c, c)$ are comparable, then $R(c, c)$ is between $R(a, b)$ and $R(b, a)$, with respect to the order in P.
Thus, the class $\{R(a, a) \mid a \in P\}$ is a kind of equilibrium for R.

Corollary

Let R be a P-valued preference relation on X, where P is a bounded chain. Then, for all $a, b, c \in X$, $a \neq b$,

$$R(a,b) \leq R(c,c) \leq R(b,a) \text{ or } R(b,a) \leq R(c,c) \leq R(a,b) \text{ and}$$

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A P-valued preference has a particular impact on the structure of membership values.
For a P-valued relation $R : X \times X \to P$, by $\mathrm{Ran}(R)$ we denote the range of R, i.e., the sub-poset of P, consisting of membership values under R:

$$\mathrm{Ran}(R) = \{p \in P \mid p = R(a,b) \text{ for some } a, b \in X\}.$$

Proposition

Let R be a P-valued preference relation on X with values in a partially ordered set P. For $a, b \in X$, define

$$R(a,b)^{\perp} := R(b,a).$$

Then, $^{\perp}$ is an order reversing involution on a sub-poset $\mathrm{Ran}(R)$ of P. In addition, for any $a, b \in X$, $R(a,b) \vee R(a,b)^{\perp}$ exists and for non-comparable $R(a,b)$ and $R(a,b)^{\perp}$,

$$R(a,b) \vee R(a,b)^{\perp} = 1.$$

Corollary

If R is a P-valued preference relation on X, then the sub-poset $\mathrm{Ran}(R) \cup \{0,1\}$ of P is a weakly orthocomplemented poset under the unary operation $^{\perp}$, defined by $R(a,b)^{\perp} = R(b,a)$, and with constant 0 and 1 being respectively the bottom and the top of P, so that $0^{\perp} = 1$.

Representation theorem

Remark

By the definition of the operation $^{\perp}$ on $\mathrm{Ran}(R)$, for every $a \in X$ we have $R(a,a)^{\perp} = R(a,a)$. Hence, for each $p \in \mathrm{Ran}(R)$ which is a value of $R(a,a)$ for some $a \in X$, we have $p^{\perp} = p$, i.e., p should be a *fixed point* of this operation on $\mathrm{Ran}(R)$.

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Theorem

Let $(P, \leq)$ be a bounded poset with the top 1 and the bottom 0, and $X \neq \emptyset$. Let $R : X \times X \to P$ be a P-valued relation. Then, R is a P-valued preference relation on X if and only if there is an order reversing involution $^\perp$ on $Q = \mathrm{Ran}(R) \cup \{0, 1\}$, such that the $(Q, \leq, {}^\perp, 0, 1)$ is a weakly orthocomplemented poset, fulfilling the following:

$$\text{If } R(x,y) = p \text{ for some } x, y \in X, \text{ then } R(y,x) = p^\perp.$$

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As a consequence, we have a characterization theorem for posets having an order reversing involution with fixed points, in terms of poset valued preferences.

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Theorem

Let P be a bounded poset such that there exists a P-valued preference relation R with $\mathrm{Ran}(R) = P$. Then, there is a unary operation $^\perp$ on P under which P is a weakly orthocomplemented poset with a nonempty set of fixed points.

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Following the case of $[0,1]$-valued relations, we introduce a particular P-valued preference relation.

Representation theorem

Let X be a nonempty set and let $(P, \leq, ^{\perp}, 0, 1)$ be a weakly orthocomplemented poset. A mapping $R : X \times X \to P$ is a P-**valued probabilistic preference relation** on X if for any $a, b \in X$ we have that

$$R(b, a) = R(a, b)^{\perp}.$$

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From the above theorems, it is straightforward that this notion is equivalent to the previous one. Namely, we have the following.

Corollary

Let X be a nonempty set (universe of objects) and let $(P, \leq, ^{\perp}, 0, 1)$ be a bounded poset with a unary operation $^{\perp}$. Then, the mapping $R : X \times X \to P$ is a poset-valued probabilistic preference relation on $Ran(R) \cup \{0, 1\}$ if and only if it is a poset-valued preference relation on $Ran(R) \cup \{0, 1\}$.

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Thus, a reciprocal relation on $[0, 1]$ is a $[0, 1]$-valued preference relation with the order reversing involution: $x^{\perp} = 1 - x$. Therefore, the concept introduced here over poset generalizes the classical notion.

Cuts of poset-valued preferences

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Theorem

Let X be a finite nonempty set. Then, there is a poset P and a poset valued preference relation $R : X \times X \to P$, such that every relation on X is a cut of R.

Cuts of poset-valued sets

The following theorem gives necessary and sufficient conditions under which a family of subsets on a set is a family of cuts of a poset-valued set.

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Theorem

Let $\mathcal{F}$ be a family of subsets of a nonempty set X and let $(P, \leqslant)$ be a poset. Then, there is a poset valued set $\mu : X \longrightarrow P$ such that $\mathcal{F}$ is its collection of cuts if and only if the following are satisfied:

1. *$\mathcal{F}$ is closed under centralized intersections and $\bigcup \mathcal{F} = X$.*
2. *There is an isotone function $E : Y \longrightarrow P$ from the poset $(Y, \supseteq)$ to $(P, \leqslant)$, where*
   $Y = \{Z_x \mid x \in X$ and $Z_x = \bigcap\{f \in \mathcal{F} \mid x \in f\}\}$,
   such that for every $r \in P$, $\bigcup E^{-1}(\uparrow r \cap E(Y)) \in \mathcal{F}$,
   and the mapping $P \longrightarrow \mathcal{F}$, defined by
   $\Phi(r) = \bigcup E^{-1}(\uparrow r \cap E(Y))$ is 'onto'.

Poset valued intuitionistic sets

Let X be a nonempty set and P be an arbitrary poset. Let $\mu : X \longrightarrow P$ and $\nu : X \rightarrow P$ be two functions from X to P.

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Let X be a nonempty set and P be an arbitrary poset. Let $\mu : X \longrightarrow P$ and $\nu : X \rightarrow P$ be two functions from X to P. Then (P, μ, ν) is **a poset valued intuitionistic set** on set X if for each $x \in X$,

$$\mu(x) \downarrow \cap \nu(x) \downarrow \subseteq S,$$

where S is $\{B\}$ if P has the bottom element B, and $S = \emptyset$ otherwise.

Poset valued intuitionistic sets

Let X be a nonempty set and P be an arbitrary poset. Let $\mu : X \longrightarrow P$ and $\nu : X \to P$ be two functions from X to P.
Then (P, μ, ν) is **a poset valued intuitionistic set** on set X if for each $x \in X$,

$$\mu(x)\downarrow \cap \nu(x)\downarrow \subseteq S,$$

where S is $\{B\}$ if P has the bottom element B, and $S = \emptyset$ otherwise.
The function μ is called the **membership function**, where $\mu(x)$ represents belonging of an element x to the intuitionistic set and the function ν is called the **non-membership function**, $\nu(x)$ representing non-belonging of an element x to the poset valued intuitionistic set.

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For each $p \in P$, two types of cut sets of (P, μ, ν) are defined as follows:

$$\mu_p = \{x \in X \mid \mu(x) \geq p\} \quad \text{and} \quad \nu_p = \{x \in X \mid \nu(x) \leq p\}.$$

Theorem

Let X be a non-empty set, $(P, \leq)$ a finite poset with the set of join-irreducible elements $\mathcal{J}$, represented by a subset of $\{0,1\}^{\mathcal{J}}$ and let $\mu : X \to P$ and $\nu : X \to P$ be two arbitrary functions. Then (P, μ, ν) is a poset valued intuitionistic set on set X if and only if $\mu(x)(i) + \nu(x)(i) \leq 1$, for all $x \in X$ and all $i \in \mathcal{J}$.

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Thank you for your attention!