The proof of CSP Dichotomy Conjecture for 5-element domain

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Outline

1. What is CSP?
2. Transitive Closure
3. 1-Consistency
4. Absorbtion
5. Rosenberg Completeness Theorem
6. Central Relations
7. Partial Order Relations
8. All Functions
9. Linear Case
10. Algorithm

Definitions

Let A be a finite set.
A mapping $A^n \to \{0,1\}$ is called an n-ary predicate.
A subset $\rho \subseteq A^n$ is called an n-ary relation.

- We do not distinguish between predicates and relations.

Let G be a finite set of predicates.

CSP(G)

Given: a conjunction of predicates, i.e. a formula

$$\rho_1(x_{i_{1,1}},\dots,x_{i_{1,n_1}}) \wedge \dots \wedge \rho_s(x_{i_{s,1}},\dots,x_{i_{s,n_s}}),$$

where $\rho_1,\dots,\rho_s \in G$.
Decide: whether the formula is satisfiable.

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Example

$A = \{0, 1, 2\}, G = \{x < y, x \leq y\}$.
CSP instances:
$x_1 < x_2 \wedge x_2 < x_3 \wedge x_3 < x_4$,

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$x_1 \leq x_2 \wedge x_2 \leq x_3 \wedge x_3 \leq x_1$, $x_1 = x_2 = x_3 = 0$.

A weak near unanimity operation (WNU) is an operation f satisfying
$f(x, x, \ldots, x) = x$ and
$f(x, \ldots, x, y) = f(x, \ldots, x, y, x) = \cdots = f(y, x, \ldots, x)$.

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Conjecture

CSP(G) is solvable in polynomial time if there exists a WNU preserving G, CSP(G) is NP-complete otherwise.

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Challenge

Given a finite set of predicates G and a WNU w that preserves G. Find an algorithm that solves CSP(G) in polynomial time.

CSP Instance

Given a CSP instance

$$\rho_1(x_{i_{1,1}}, \ldots, x_{i_{1,n_1}}) \wedge \cdots \wedge \rho_s(x_{i_{s,1}}, \ldots, x_{i_{s,n_s}}),$$

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where $\rho_1, \dots, \rho_s \in G$.

Step 1: Generate all binary constraints

For every constraint $\rho(x_1, \dots, x_n)$ and $i, j \in \{1, 2, \dots, n\}$ we add a binary constraint $\sigma_{i,j}(x_i, x_j)$, where

$$\sigma_{i,j}(y_i, y_j) = \exists y_1 \dots \exists y_{i-1} \exists y_{i+1} \dots \exists y_{j-1} \exists y_{j+1} \dots \exists y_n \, \rho(y_1, \dots, y_n).$$

Step 2: Transitive closure.

For every 2 binary constraints $\rho_1(x_i, x_j)$ and $\rho_2(x_j, x_k)$ we add the constraint $\rho_3(x_i, x_k)$, where $\rho_3(y_1, y_2) = \exists z \rho_1(y_1, z) \wedge \rho_2(z, y_2)$.

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Example

We have constraints $(x_1 \leq x_2)$ and $(x_2 \leq x_3)$.
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Step 3: Constraint propagation.

We can provide 1-consistency:
if a variable x_i takes only values from $D_i' \subsetneq D_i$ in a constraint then we reduce the domain of x_i to D_i' and restrict all other constraints.

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$x_1 < x_2 \Rightarrow$ the domain of x_2 can be reduced to $\{1, 2\}$,

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CSP instance on $A = \{0, 1, 2\}$, $x_1 < x_2 \wedge x_2 < x_3 \wedge x_3 < x_4$.
$x_1 < x_2 \Rightarrow$ the domain of x_2 can be reduced to $\{1, 2\}$,
$x_2 < x_3 \Rightarrow$ the domain of x_3 can be reduced to $\{2\}$,
$x_3 < x_4 \Rightarrow$ no solution for x_4. We get a contradiction.

- We cannot reduce forever, hence either we get 1-consistency, or we get a contradiction.

Absorbtion

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Definition

A subuniverse B absorbs A if there exists a binary operation $f \in \mathrm{Clo}(w)$ such that $f(B, A) \subseteq B$ and $f(A, B) \subseteq B$.

- $\mathrm{Clo}(w)$ is the clone generated by a WNU w.

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Step 4: Absorbing restriction.

If B_i absorbs D_i, we reduce the domain D_i to B_i.
Then we go to Step 3 and provide 1-consistency!

- Constraint propagation cannot give a contradiction in this case!

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2. Rosenberg's description of all maximal clones on k elements

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Main Results in Clone Theory

1. ~~The description of all clones on 2 elements (Post's Lattice).~~
2. Rosenberg's description of all maximal clones on k elements

Rosenberg Completeness Theorem

There are only following maximal clones on k elements.

1. Maximal clone defined by a unary relation;
2. Maximal clone of monotone functions;
3. Maximal clone of autodual functions;
4. Maximal clone defined by an equivalence relation;
5. Maximal clone of quasi-linear functions;
6. Maximal clone defined by a central relation;
7. Maximal clone defined by an h-universal relation.

Let C be the clone generated by the WNU w on D_i and all constants from D_i.

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5. ~~Maximal clone defined by an equivalence relation;~~ factorize WNU, generate a new clone, apply Rosenberg Theorem again
6. Maximal clone defined by a central relation;
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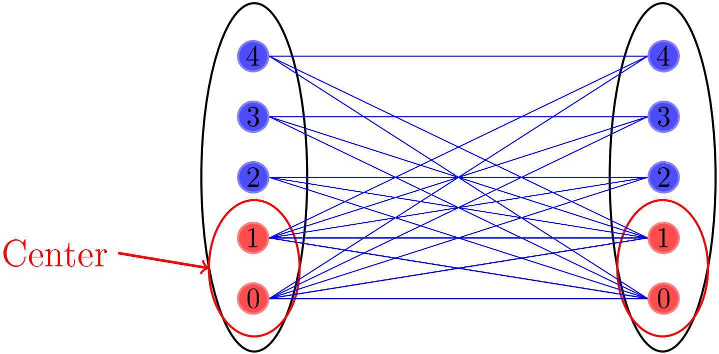
8. Maximal clone of quasi-linear functions;

To simplify we consider only binary central relations.

A relation $\rho \subseteq A \times A$ is called central if it is reflexive, symmetric, and there exists c such that $\{c\} \times A \subseteq \rho$.

- the set of all elements c such that $\{c\} \times A \subseteq \rho$ is called center.



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If C_i is a center in D_i, we reduce D_i to C_i.
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If C_i is a center in D_i, we reduce D_i to C_i.
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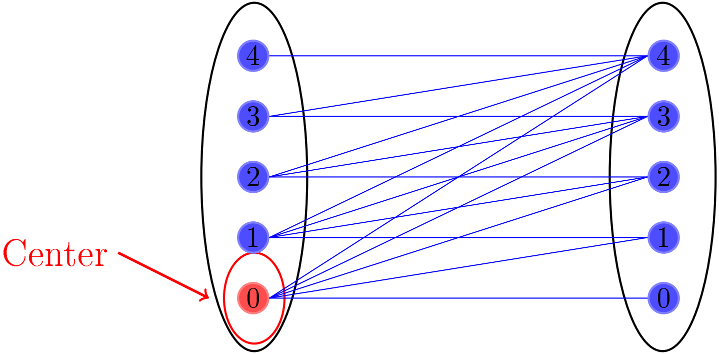
- if we don't have binary absorbtion, then constraint propagation cannot give a contradiction in this case!

Every maximal clone of monotone functions is defined by a partial order relation with a greatest and a least element.

- the least element can be viewed as a center.



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Maximal clone of quasi-linear functions

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2. Factorize all the constraints, i.e. replace every predicate ρ by

$$\rho'(x_1, \ldots, x_n) = \exists y_1 \ldots \exists y_n \, \rho(y_1, \ldots, y_n) \wedge (x_1, y_1) \in \sigma_{i_1} \wedge \cdots \wedge (x_n, y_n) \in \sigma_{i_n}$$

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5. If Θ doesn't have a solution then we find a subset A_i' of the original domain A_i such that no solutions with $x_i \in A_i' \setminus A_i$.

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- Either it gives a solution,
- or it reduces the original domain A_i to A_i',
- or it proves that no general solutions.

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Theorem

If an algebra $\mathbb{A}$ omits unary type and affine type, then Steps 1-3 of the algorithm solve $CSP(\mathbb{A})$.

Thank you for your attention